FINANCIAL MARKETS DURING ECONOMIC CRISIS

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Abstract

Contemporary financial depression of the financial markets proves high fluctuations of the prices of the stocks. These fluctuations have considerable impact on the values of the financial portfolios. Classical approaches to modeling of the behavior of the prices of the stocks may produce wrong predictions of their future values. That is the reason why we introduce in this paper the fractal market analysis. Fractal structure accepts global determinism and local randomness of the behavior of the financial time series. We will use R/S analysis in this paper. R/S analysis can distinguish fractals from other types of time series, revealing the self-similar statistical structure.

Key words: financial time series modeling, fractal, R/S analysis, Hurst exponent

1. Introduction

The financial markets are an important part of any economy. For that reason, financial markets are also an important aspect of every model of the economy[1]. Markets are "efficient" if prices reflect all current information that could anticipate future events. Therefore, only the speculative, stochastic component could be modeled, the change in prices due to changes in value could not. If markets do not follow a random walk, it is possible that we may be over-or understanding our risk and return potential from investing versus speculating.

2. Introduction to Fractals and the Fractal dimensions

The development of fractal geometry has been one of the 20-th century's most useful and fascinating discoveries in mathematics ([2], p.45). Fractals give structure to complexity, and beauty to chaos. Most natural shapes, and time series, are best described by fractals. Fractals are self–referential, or self–similar. Fractal shapes show self–similarity with respect to space. Fractal time series are random fractals, which have more in common with natural objects than the pure mathematical fractals we will cover initially. We will be concerned primarily with fractal time series, but fractal shapes give a good intuitive base for what "self-similarity" actually means. Figure 1 shows daily and weekly Bank of America Corporation prices[2] for consecutive observations from march 2007 to may 2009. With no scale on the *X* and *Y* axes, we are not able to determine which graph is which. Figure 1 illustrates self-similarity in a time series.

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[1] http://www.economymodels.com/financialmarkets.asp
[2] Data were retrieved from www.yahoo.com

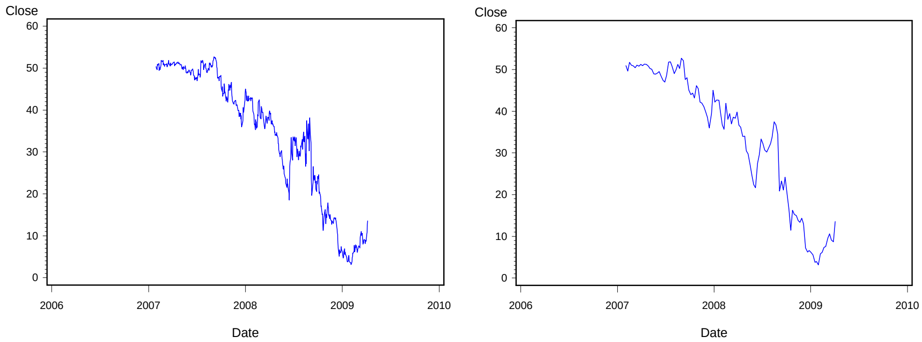

Figure 1: Daily and weekly prices of stock Bank of America Corp.

Fractal shapes can be generated in many ways. The simplest way is to take a generating rule and iterate it over and over again. Random fractals are combination of generating rules chosen at random for different scales. Combination of randomness coupled with deterministic generation rules, or "causality", can make fractals useful in capital market analysis. Random fractals ([2], p.51) do not necessarily have pieces that look like pieces of the whole. Instead, they may be qualitatively related. In the case of time series, we will find that fractal time series are qualitatively self similar in that, at different scales, the series have similar statistical characteristics. If we would like to understand the underlying causality of the structure of time series, then classical geometry offers little help. May be, time series is a random walk – a system so complex that the prediction becomes impossible. In statistical term, the number of degrees of freedom or factors influencing the system is very large. These systems are not well-described by standard Gaussian statistics. Standard statistical analysis begins by assuming that the system under study is primarily random; that is, the causal process that created the time series has many component parts, or degree of freedom, and the interaction of those components is so complex that deterministic explanation is not possible ([1], p.53). Only probabilities can help us to understand and take advantage of the process. The underlying philosophy implies that randomness and determinism cannot coexist. In order to study the statistics of these systems and create a more general analytical framework, we need a probability theory that is nonparametric. In this paper we introduce nonparametric methodology that was discovered by H.E. Hurst[3].

In advance, we introduce the term fractal dimension. The fractal dimension describes how a time series fills its space, is the product of all factors influencing the system that produces time series ([2], p.57). Fractal time series can have fractional dimensions. The fractal dimension of a time series measures how jagged the time series is ([1], p.16). As would be expected, a straight line has a fractal dimension of 1. Time series is only random when it is influenced by a large number of events that are equally likely to occur. In statistical term, it has a high number of degree of freedom. A random series would have no correlation with previous points. Nothing would keep the points in the same vicinity, to preserve their dimensionality. Instead, they will fill up whatever space they are placed in. A nonrandom time series will reflect the nonrandom nature of its influences. The data will clump together, to reflect the correlations inherent in its influences. In other words, the time series will be fractal. To determine the fractal dimension, we must measure how the object clumps together in its space. However, a random walk has 50–50 chance of rising or falling, hence, its fractal dimension is 1.50. The fractal dimension of a time series is important because it recognizes that process can be somewhere between deterministic (a line with fractal dimension of 1) and random (a fractal dimension of 1.50). In fact, the fractal dimension of a line can range from 1

[3] Hurst, H.E. 1951. The Long-Term Storage Capacity of Reservoirs. In: Transaction of the American Society of Civil Engineers 116.

to 2. The normal distribution has an integer dimension of 2, which many of characteristics of the time series. At values $1.50<d<2$, a time series is more jagged than a random series.

They are many ways of calculating fractal dimensions. We introduce methodology of the Hurst exponent H, and we convert it into the fractal dimension d in this paper.

3. R/S analysis and Hurst exponent

Hurst was aware of Einstein`s[4] work of Brownian motion. Brownian motion became the primary model for a random walk process. Einstein found that the distance that a random particle covers increases with the square root of time used to measure it, or:

$$R=T^{0.50}, \qquad (1)$$

where R is the distance covered and T is a time index.
Equation (1) is called the *T* to *the one–half rule*, and it is commonly used in statistics. Financial economists use it to annualize volatility or standard deviation. To standardize the measure over time, Hurst decided to create a dimensionless ratio by dividing the range by the standard deviation of the observations. Hence, the analysis is called rescaled range analysis (R/S analysis). Hurst found that most natural phenomena follow a "biased random walk" – a trend with noise. The strength of the trend and the level of noise could be measured by how the rescaled range scales with time, that is, by how high H is above 0.50. Peters ([1] p.56) reformulated Hurst`s work for a general time series as follows.

We begin with a time series, $X=\{x_1, x_2, \ldots, x_n\}$, to represent n consecutive values. For markets, it can be the daily changes in price of a stock index. The rescaled range was calculated by first rescaling or "normalizing" the data by subtracting the sample mean x_m:

$$Z_r=(x_r–x_m), \qquad r=1,2,\ldots,n \qquad (2)$$

The resulting series, Z, now has a mean of zero. The next step creates a cumulative time series Y:

$$Y_1=(Z_1+Z_r), \qquad r=2,3,\ldots,n \qquad (3)$$

Note that, by definition, the last value of Y (Y_n) will always be zero because Z has a mean of zero. The adjusted range, R_n, is the maximum minus minimum value of the Y_r:

$$R_n=max(Y_1, Y_2,\ldots, Y_n)–min(Y_1, Y_2,\ldots, Y_n). \qquad (4)$$

The subscript, n, for R_n now signifies that this is the adjusted range for $x_1, x_2, \ldots, x_n$. Because Y has been adjusted to a mean of zero, the maximum value of Y will always be greater than or equal to zero, and the minimum will always be less than or equal to zero. Hence, the adjusted range, R_n, will always be nonnegative. This adjusted range, R_n, is the distance that the system travels for time index n. If we set $n=T$, we can apply equation (1), provided that the time series, X, is independent for increasing values of n. However, equation (1) applies only to time series that are in Brownian motion (they have zero mean, and variance is equal to one). To apply this concept to time series that are not in Brownian motion, we need to generalized equation (1) and take into account systems that are not independent. Hurst found that the following was a more general form of equation (1):

$$(R/S)_n=c.n^H \qquad (5)$$

The subscript, n, for $(R/S)_n$ refers to the R/S value for $x_1, x_2, \ldots, x_n$ and c is a constant.

The R/S value of equation (5) is referred to as the *rescaled range* because it has zero mean and is expressed in terms of local standard deviation. In general, the R/S value scaled as we increase the time increment, n, by a power–law value equal to H, generally called the *Hurst exponent*.

Rescaling allows us to compare periods of time that may be many apart. In comparing stock returns of the 1920s with those of the 1980, prices present a problem because of inflationary growth. Rescaling minimize this problem, by rescaling the data to zero mean and standard deviation of one, to allow diverse phenomena and time periods to be compared.

[4] Einstein, A. 1908. Uber die von der molekularkinetischen Theorie der Warme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen. Annals of Physics 322.

Rescaled range analysis can also describe time series that have no characteristic scale. This is a characteristic of fractals.

The Hurst exponent can be approximated by plotting the log(R/S_n) versus the log(n) and solving for the slope through an ordinary least squares regression:

$$log(R/S_n)=log(c)+H.log(n) \tag{6}$$

If a system is independently distributed, then H=0.50. When H differed from 0.50, the observations are not independent. Each observation carried a „memory“ of all the events that preceded it. What happens today influences the future. Where we are now is a result of where we have been in the past. Time is important. The impact of the present on the future can be expressed as a correlation:

$$C=2^{(2H-1)}-1, \tag{7}$$

where C is correlation measure and H is Hurst exponent.

It is important to remember that this correlation measure is not related to the Auto Correlation Function (ACF) of Gaussian random variables ([2], p.70). The ACF assumes Gaussian, or near-Gaussian, properties in the underlying distribution. The ACF works well in determining short-run dependence, but tends to understate long-run correlation for non-Gaussian series (full mathematical explanation we find in [5].

There are three distinct classifications for the Hurst exponent ([2], p.64):

1. H=0.50: time series is random, events are random and uncorrelated. Equation (7) equals zero. The present does not influence the future. Its probability density function can be normal curve, but it does not have to be. R/S analysis can classify an independent series, no mater what the shape of the underlying distribution.
2. $0 \le H < 0.50$: time series is antipersistent, or ergodic. If the time series has been up in the previous period, it is more likely to be down in the next period. Conversely, if it was down before, it is more likely to be up in the next period. The strength of this antipersistent behavior depends on how close H is to zero. The closer it is to zero, the closer C in equation (7) moves toward –0.50, or negative correlation. This time series is more volatile than a random series.
3. $0.50 \le H < 1.00$: time series have a persistent or trend-reinforcing character. If the series has been up (down) in the last period, then the chances are that it will continue to be positive (negative) in the next period. Trend is apparent. The strength of the trend-reinforcing behavior, or persistence, increases as H approaches 1.0. The closes H is to 0.5, the noisier it will be, and the less defined its trends will be. Persistent series are fractional Brownian motion, or biased random walk[5]. The strength of the bias depends on how far H is above 0.50. A high H value shows less noise, more persistence and clearer trends than do lower value. A high H means less risk.

4. Testing R/S analysis

To evaluate the significance of R/S analysis, we calculate expected value of the R/S statistics and the Hurst exponent. We compare the behavior of our process, described by R/S analysis with an independent and random system and gauge its significance.

We will test this null hypothesis: “The process is independent, identically distributed and is characterized by a random walk”[6].

To verify this hypothesis, we calculate expected value of the adjusted range[7] $E(R/S_n)$ and its variance[8] $Var(E(R/S_n))$.

[5] Biased random walks were extensively studied by Hurst in the 1940s and again by Mandelbrot in the 1960s and 1970s. Mandelbrot called them fractional brownian motions.([2], p.61)

[6] This process has Gaussian structure (see [1], p.66).

[7] This formula was derrived by Anis and Lloyd ([1], p.71)

[8] Variance was calculated by Feller ([1], p.66)

$$E(R/S_n) = \frac{n-0.5}{n} \cdot \left(n \cdot \frac{\pi}{2}\right)^{-0.5} \sum_{r=1}^{n-1} \sqrt{\frac{(n-r)}{r}} \qquad (8)$$

$$Var(E(R/S_n)) = \left(\frac{\pi^2}{6} - \frac{\pi}{2}\right) \cdot n\,. \qquad (9)$$

Using the results of equation (8) we can generate expected values of the Hurst exponent. The expected Hurst exponent will vary depending on the values of n we use to run the regression. Any range will be appropriate as long as the system under study and the $E(R/S_n)$ series cover to the same values of n. For financial purpose, we will begin with n=10. The final value of n will depend on the system under study.

R/S values are random variables, normally distributed and therefore we would expect that the values of H would also be normally distributed (see Peters [1], p.72):

$$Var(H_n) = \frac{1}{T}, \qquad (10)$$

where T is total number of observations in the sample. Note that the $Var(H_n)$ does not depend on n or H, but it depends on the total sample size T.
Now t-statistics will be used to verify of the significance of the null hypothesis.

5. Finding Cycles

Hurst[9] was the first to realize that an underlying periodic component could be detected with R/S analysis ([1], p. 88) and used simple statistic to test stability. Using this statistic we give a more precise measure of the cycle length. The statistics is called V and it is defined as follows ([1], p.92):

$$V_n = \frac{(R/S)_n}{\sqrt{n}} \qquad (11)$$

This ratio would result in a horizontal line if the R/S statistics was scaling with the square root of time. In other words, a plot of V versus log(n) would be flat if the process was an independent, random process. If the process was persistent and R/S was scaling at a faster rate than the square root of time (H>0.50), then the graph would be upwardly sloping. Conversely, if the process was antipersistent (H<0.50), the graph would be downward sloping. By plotting V on the vertical axis and log(n) on the horizontal axis, the "breaks" would occur when the V chart flattens out. At those points, the long-memory process has dissipated. R/S analysis is capable of determining periodic cycles, even when they are superimposed. The real power of R/S analysis is in finding nonperiodic cycles.

6. Empirical study

We apply R/S analysis to the daily and weekly closing stock prices Bank of America from 29.05.1986 to 7.5.2009 and the data follow from www.yahoo.finance.com (see Figure 2). R/S analysis needs a long time intervals. We have 5787 observations for daily frequency (only trading days) and 1197 observations for weekly frequency.

When analyzing markets, we use logarithmic returns, defined as follows:

$$S_t = ln(P_t/P_{t-1}), \qquad (8)$$

where S_t is logarithmic return at time t and P_t is stock price at time t.

[9] Hurst, H. E. 1951. *The Long-Term Storage Capacity of Reservoirs*. Transactions of the American Society of Civil Engineers 116.

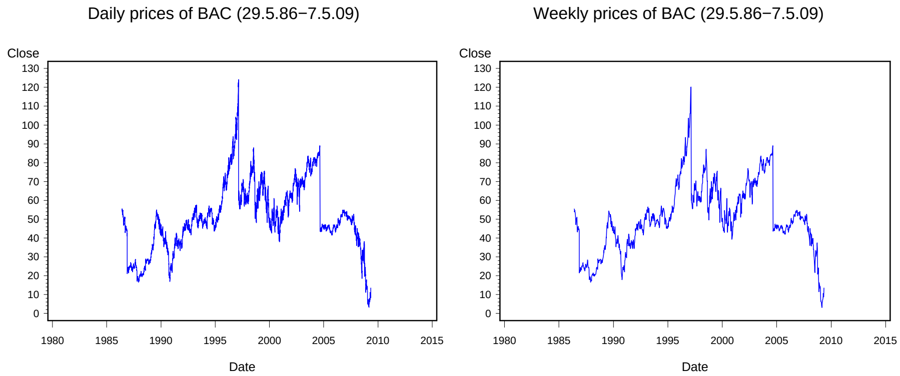

Figure 2: Daily and weekly prices of Bank of America Corp. from 29.5.86 to 7.5.09

For R/S analysis, logarithmic returns are more appropriate than the more commonly used percentage change in prices. The range used in R/S analysis is the cumulative deviation form the average, and logarithmic returns sum to cumulative return, while percentage changes do not (see [2], p.83).

We will examine the behavior of H over different time increments, for daily and weekly returns of stock Bank of America Corp. (BAC).

Table 1 show both the R/S_n values and the V_n. Figure 3 (on the left) shows the *log R/S* plot for daily return data for T=5775 observations. Also plotted is $E(R/S)$ (calculated using equation (8)) as a comparison against the null hypothesis that the system is an independent process.

The regression yielded H=0.53540 and $E(H)$=0.56213 for daily returns (see Table 3). The variance of $E(H)$, as shown in equation (10) is 0.0002, for Gaussian random variables. The standard deviation of $E(H)$ is 0.0132. The H value for daily returns is –2.0313 standard deviation bellow its expected value, a significant result. The regression yielded H=0.53520 and $E(H)$=0.56952 for weekly returns (see Table 4). The variance of $E(H)$ is 0.0003 and standard deviation of $E(H)$ is 0.0132. The H value for weekly returns is –1.8418 standard deviation bellow its expected value, a non significant result for confidence level α=0.05, it means that weekly returns are independent, identically distributed and they are characterized by a random walk.

We see a systematic deviation from the expected values on the Figure 3. However, a break in the *R/S* graph appears to be at n=68 observations (*log*(68)≈4,22), for n=340 observations (*log*(340)≈5,83) and for n=1445 observations (*log*(1445)≈7,28). To estimate precisely where this break occurs, we calculate the V-statistics using equation (11) (V-statistics versus $log(n)$ is plotted in right Figure 3). V-statistics is decreasing from V_{68}=1.13 to V_{85}=1.11. Hurst exponent were estimated from the R/S plot and the E(R/S) and H equals to 0.57486 and expected H equals to 0.63978, for $10 \leq n < 70$. Hurst exponent equals to 0.54422 for $70 < n \leq 2890$. The series exhibits persistence (H>0.50). The next subperiod is $70 < n \leq 1445$, where the slope appeared to follow the E(R/S) line. H=0.51900 and $E(H)$= 0.52939 and they are excessively closely and therefore *H is* insignificant. Process became persistent.

Figure 4 and Table 2 show the results of R/S analysis. Unfortunately, the Hurst exponent is not significant. H=0.53520 and $E(H)$= 0.56952 (see Table 4). The Hurst exponent is –1.8418 standard deviations bellow its expected value. We need 3396 observations to achieve significance[10]. Unfortunately, we have only T=1188 observations for weekly return data of stock Bank of America. Stocks Bank of America were not come off until 1986, we cannot increase the time frame.

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[10] We need $T=4/(H-E(H))^2$ points, see ([1], p.153)

N	log N	log R/S	R/S	E(R/S)	V statistic	
					BAC	E(R/S)
10	2,302585	1,122582	3,072779	2,650278	0,971698	0,838092
17	2,833213	1,463863	4,322625	3,879877	1,048391	0,941009
20	2,995732	1,546395	4,694515	4,324742	1,049725	0,967042
34	3,526361	1,852481	6,37562	6,050077	1,09341	1,03758
68	4,219508	2,231924	9,317779	9,101265	1,129947	1,10369
85	4,442651	2,325834	10,23522	10,32771	1,110165	1,120197
170	5,135798	2,701515	14,90229	15,13091	1,142953	1,160488
289	5,666427	2,978472	19,65775	20,10602	1,156338	1,182707
340	5,828946	3,057879	21,28237	21,94454	1,154199	1,19011
578	6,359574	3,347838	28,44118	28,96639	1,182997	1,204843
1156	7,052721	3,68908	40,00803	41,44743	1,176707	1,219042
1445	7,275865	3,796379	44,5396	46,47719	1,171689	1,222661
2890	7,969012	4,307041	74,22053	66,21137	1,380623	1,23164

Table 1: R/S analysis and V-statistics, Bank of America: daily returns

N	log N	log R/S	R/S	E(R/S)	V statistic	
					BAC	E(R/S)
11	2,397895	1,166956	3,212199	2,848343	0,968515	0,858808
12	2,484907	1,186805	3,276594	3,037391	0,945871	0,876819
18	2,890372	1,433343	4,192693	4,032329	0,988227	0,950429
22	3,091042	1,532168	4,628198	4,602551	0,986735	0,981267
27	3,295837	1,63842	5,147031	5,245172	0,990546	1,009434
33	3,496508	1,73145	5,648838	5,940635	0,983337	1,034132
36	3,583519	1,818525	6,162762	6,264156	1,027127	1,044026
44	3,78419	1,890857	6,625041	7,065256	0,998763	1,065127
54	3,988984	2,07206	7,941168	7,968737	1,080656	1,084408
66	4,189655	2,188393	8,920869	8,947224	1,098083	1,101327
99	4,59512	2,373262	10,73235	11,2472	1,078642	1,130386
108	4,682131	2,371679	10,71537	11,80396	1,031087	1,135837
132	4,882802	2,551656	12,82832	13,18354	1,116562	1,14748
198	5,288267	2,868894	17,61753	16,4285	1,252023	1,167522
297	5,693732	2,975899	19,60723	20,39936	1,137727	1,183691
396	5,981414	3,195528	24,42308	23,77524	1,227306	1,194751
594	6,386879	3,502434	33,19614	29,3806	1,362054	1,2055

Table 2: R/S analysis and V-statistics, Bank of America: weekly returns

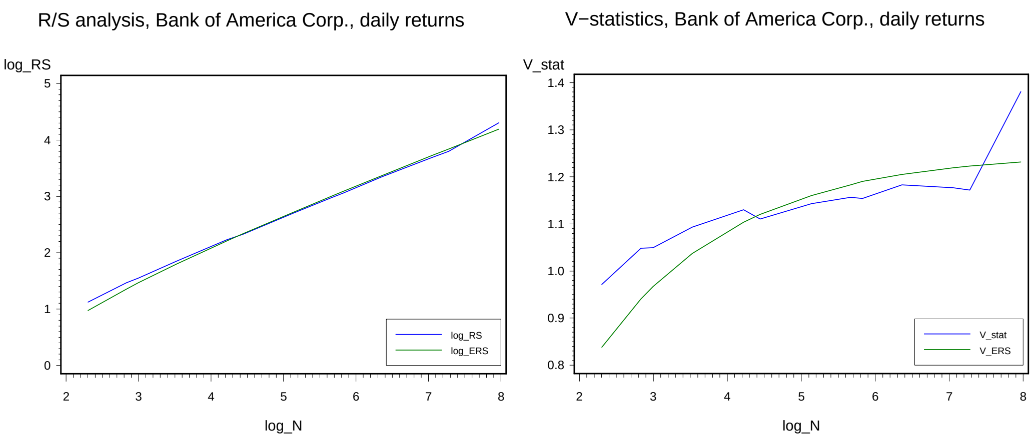

Figure 3: R/S analysis and V-statistics, Bank of America: daily returns

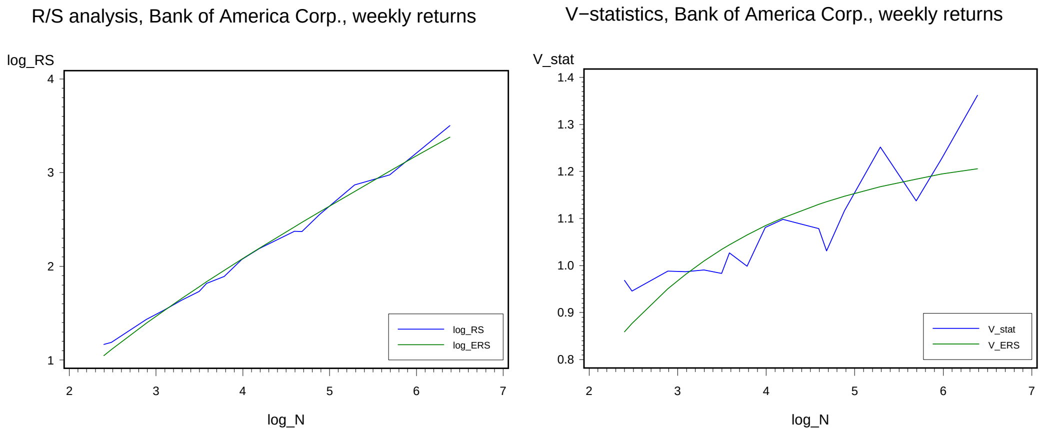

Figure 4: R/S analysis and V-statistics, Bank of America: weekly returns

Parameter Estimates for daily returns of BAC					
	DF	**Parameter Estimate**	**Standard Error**	**t Value**	**Pr > |t|**
Intercept	1	-0,05695	0,01963	-2,90	0,0099
Hurst exponent *H*	1	0,53540	0,00393	136,16	<.0001
R-Square for *H*	0,9991				
Adj R-Sq for *H*	0,9990				
Expected Intercept	1	-0,20089	0,03168	-6,34	<.0001
expected Hurst exponent *E*(*H*)	1	0,56213	0,00635	88,58	<.0001
R-Square for *E(H)*	0,9978				
Adj R-Sq for *E(H)*	0,9977				
Number of Observations	19				
Var(*E*(*H*))	0,0002				
s(***E***(H))	0,0132				
significance	-2,0313				

Table 3: Hurst exponent for R/S analysis, Bank of America: daily returns

Parameter Estimates for weekly returns of BAC					
	DF	**Parameter Estimate**	**Standard Error**	**t Value**	**Pr > |t|**
Intercept	1	-0,02633	0,03035	-0,87	0,3922
Hurst exponent *H*	1	0,53520	0,00647	82,72	<.0001
R-Square for *H*	0,9955				
Adj R-Sq for *H*	0,9953				
Expected Intercept	1	-0,23026	0,02350	-9,80	<.0001
expected Hurst exponent *E*(*H*)	1	0,56952	0,00501	113,66	<.0001
R-Square for *E(H)*	0,9976				
Adj R-Sq for *E(H)*	0,9975				
Number of Observations	33				
Var(*E*(*H*))	0,0003				
s(***E***(H))	0,0186				
significance	-1,8418				

Table 4: Hurst exponent for R/S analysis, Bank of America: weekly returns

Now, we divide time series to three parts: from 29 may 1986 to 25 February 1997, from 26 February 1997 to 27 August 2004 and from 30 August 2004 to 6 may 2009. These dates are corresponding to the rapid changes in the prices of the stock.

From Table 5 we see that time series had persistent character (time series is fractal random walk) from 29 May 1986 to 6 May 2009 and it is significant result. Time series is random walk from 29 |May 1986 to 25 February 1997. For next period, from 26 February 1997 – 27 August 2004 time series is significantly antipersistent (time series is more volatile than a random series) and last period indicate random walk. These results may be caused by insufficient number of data.

	Estimated *H*	Expected *H*	Standard deviation	significance
Hurst coeficient 29 may 1986 -6 may 2009	0,53540	0,56213	0,013146519	-2,033237792
Hurst coeficient 29 may 1986 -25 feb 1997	0,56067	0,55842	0,019181178	0,117302494
Hurst coeficient 26 feb 1997 - 27 aug 2004	0,47579	0,56174	0,023014365	-3,734623933
Hurst coeficient 30 aug 2004 - 6 may 2009	0,54495	0,58554	0,029135827	-1,393130167

Table 5: Hurst exponent for Bank of America daily return

7. Conclusion

In this paper we have shown how it is possible to measure the impact of information on the time series by using Hurst exponent *H*([2], p.102). *H*=0.50 implies a random walk. Yesterday`s events do not impact today. Today`s events do not impact tomorrow. The events are uncorrelated. Old news has already been absorbed and discounted by the market. *H* greater than 0.50 implies that today`s events do impact tomorrow. Information received today continues to be discounted by the market after it has been received. This is not simply serial correlation; it is a longer memory function. Information can impact the future for very long periods, and it goes across time scales.

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